RESTRICTION GEOMETRIC OPERATORS TO LEAVES OF FOLIATION

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joint work with

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Abstrakt

Since 1968, when gradients were intoduced in the famous paper by Stein and Weiss *Generalization of the Caucy-Riemann equations and representations of the rotation group* [8], they developed into a large branch of global analysis or geometry, differential operators or the representation theory. Let us start then with few words introducing the notion of the gradient. The first and the most important operator of differntial geometry is the covariant derivative ∇. If one starts from any linear bunndle E over M, a differential manifold, and terminate together with ∇ in the bundle $E \otimes TM^*$ and if, additionally, one has a Lie group G acting both on E and $E \otimes TM^*$ (and such a group is always strictly associated to any geometric structure gardening on M), then one can think on splitting both E and $E \otimes TM^*$ onto direct sums of G-irreducible subspaces. Then, the restriction of covariant derivative ∇ to any one of such subbundles of E composed with the projection onto any one of $E \otimes TM^*$ is just a G-gradient. Gradients are then the simplest bricks of which the covariant derivative is build.

In the Riemanian geometry the group structure G is $O(n)$. Since we are mainly interested here in the Riemannian structure, we will consider $O(n)$-gradients only. We will call them simply gradients or, when restricted to a foliation, relative gradients. All the natural first order linear differentail operators in Riemannian geometry are either gradients or their compositions. For examle, the exterior and interior derivatives d and δ, repectively, the Cauchy-Rieman operator $\bar{\partial}$ are gradients while the classical Dirac operator on exterior forms, namely, $d + \delta$ is their sum. Gradients depend of the geometry of M (the group G) and this is obvious, but, on the other hand, they can themselves, eg. by their spectral properties, determine, to some extent, the geomety (cf. [7], [3]).

It is known that differential operators and so gradients in particular, do not restrict trivially to submanifolds or, the more so, to foliations. Recall that a foliation of a maifold M is a partition of M onto a topological sum of pairly disoint submanifolds (all of the same dimension). These submanifolds are called the leaves of the foliation. The definition and the basic theory of foliated manifods can be found in [4].

Many authors have cosidered a resonable theory of *foliated* operators such as the Laplace or the Laplace-Beltrami operators or many other natural ones under assumption that the foliation in question is Riemannian (cf. [1], [2], [5], [6]. The assumption means in practice that such a foliation is given locally by a submersion that is transversally isometric, ie. the differental of such a submersion, restricted to the orthogonal complement of its kernel, is an isometry. So, the assumption on a foliation of being Riemannian is rather restrictive. Here we make much weaker assumptionon. We work namely with so called SL(q)-foliations introduced by Tondeur in his book [9]. Roughly speaking such a foliation is given locally by a submersion that is transversally volume preserving.

We show that all the possible $O(n)$-gradients can be reasonably restricted to such foliations so that they inherit all the properties of their ancestors. Moreover, we set that their adjoints are not only formal adjoints with respect to the leaf scalar product, what should be rather expected, but also with the global scalar produt on M. Finally we show that the method works well also for other natural differential operatos like eg., the Dirac one.

Literatura

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